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THE MOBILITIES OF ELECTRONS IN AIR

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In previous publications[1,2] the writer has shown that electrons in gases at atmospheric pressure have rather high mobilities which vary with the field strength. In another paper[3] it was shown by a study of the mobility curves for ions in air at reduced pressures that the theory of negative ion formation of J. J. Thomson[4] is qualitatively adequate to explain the phenomena observed. In order to obtain a quantitative verification of this theory it becomes essential to measure the mobilities of electrons in air at pressures above 40 mm. The determination of such mobilities seemed at the outset impossible owing to the attachment of the electrons to the oxygen molecules to form ions. Investigation, however, showed that by neutralizing the field produced by an accumulation of the ions, and perhaps in a measure preventing its formation,[2] by the use of a constant retarding field superposed upon the alternating field, it was possible to measure mobilities of an electronic magnitude in air. The results which were obtained from measurements made in this manner, and whose interpretation may be somewhat obscure because of attachments, will constitute the contents of this paper.

The measurements were made on carefully dried air passed over $CaCl_2$ and P_2O_5, in an ion chamber previously described.[2] The electrical connections were just as those shown in the paper on mobilities of electrons in H_2[2] except that one and sometimes two "Eveready" flashlight batteries of three volts each were connected with their positive pole to the plate source of photoelectrons, and their negative poles to the source of alternating potential. In all these measurements the plate distance d used was 1.955 cm. As before for a given period of oscillation T, and pressure p, the potential E_0 at which the current voltage curve cut the axis was determined. When the retarding potential R applied to the plates is absent the mobility may be computed from $k = \frac{\pi}{\sqrt{2}} \frac{d^2}{E_0 T}$, and the mobility constant (i.e., the mo-

bility reduced to atmospheric pressure), is given by $K = kp/760$. The equation for k is obtained from the condition that the electrons can just cross the plate distance d, in the time of a half cycle, $T/2$, at the critical voltage E_0 as read on a static voltmeter. This condition is expressed by the following equation

$$d = k\frac{\sqrt{2}E_0}{d}\int_0^{\frac{T}{2}} \sin\frac{2\pi t}{T}\,dt$$

whence we at once obtain k as given by

$$k = \frac{\pi d^2}{\sqrt{2}E_0\frac{T}{2}\left[-\cos\frac{2\pi t}{T}\right]_0^{\frac{T}{2}}}.$$

Now the term $\frac{\sqrt{2}}{\pi}E_0\frac{T}{2}\left[-\cos\frac{2\pi t}{T}\right]_0^{\frac{T}{2}}$ is nothing but the area under a loop of the sine wave alternating potential, taken over the interval O to $T/2$. In the case where a retarding potential R is imposed on the plates, the portion of the sine curve effective in driving the electrons the distance d is that portion of the area under the sine curve lying between it and the line representing the potential R. This area is

$$\left[\frac{\sqrt{2}E_0\frac{T}{2}\left[-\cos\frac{2\pi t}{T}\right]_{\frac{T}{2\pi}(\sin^{-1}f)}^{\frac{T}{2\pi}(\pi-\sin^{-1}f)}}{\pi} - \frac{\sqrt{2}E_0 f\frac{T}{2}(\pi-2\sin^{-1}f)}{\pi}\right]$$

where $f = R/\sqrt{2}E_0$. Hence k becomes

$$k = \frac{\pi d^2}{\sqrt{2}E_0 T}\left\{\frac{2}{\left[-\cos\frac{2\pi t}{T}\right]_{\frac{T}{2\pi}(\sin^{-1}f)}^{(\pi-\sin^{-1}f)\frac{T}{2\pi}} - f(\pi-2\sin^{-1}f)}\right\}.$$

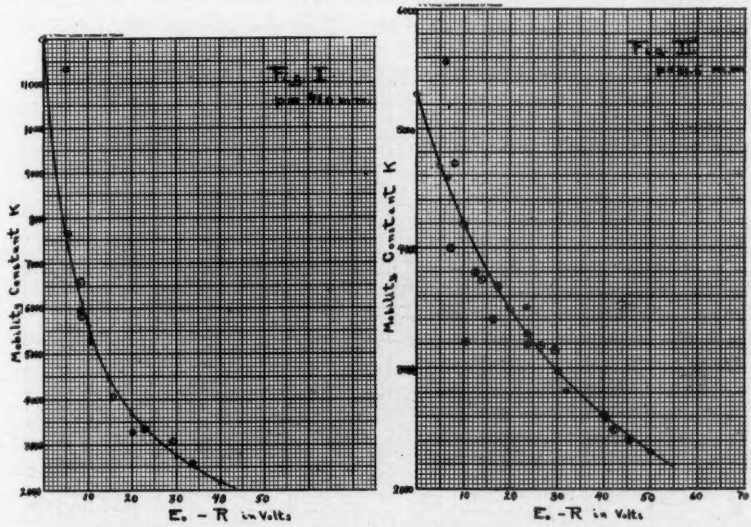
Thus K the mobility constant determined with a retarding field $R = \sqrt{2}fE_0$ is at once obtained by multiplying the value of the mobility constant K computed from $K = \frac{\pi d}{\sqrt{2}E_0 T}\left(\frac{p}{760}\right)$ by the appropriate value of the factor

$$\left\{\frac{2}{\left[-\cos\frac{2\pi t}{T}\right]_{\frac{T}{2\pi}\sin^{-1}f}^{(\pi-\sin^{-1}f)\frac{T}{2\pi}} - f(\pi-2\sin^{-1}f)}\right\}.$$

The use of this retarding field and the correction for its use just given is only permissible on the assumption that the field produced by the ions when the retarding field is on is small compared to it. Were this not true

the correction applied for R would be greater than it really should be, for the potential R does not really act alone, but is partially neutralized by the field due to the ions. That the assumption is justified for the case where $p = 41$ and $p = 51.5$ mm. is shown by the fact that the values of K obtained with retarding fields of 3 and 5.8 volts show no difference.

The values of K obtained for the range of frequencies available for pressures of 41, 51.5, 60, 66.5 and 92 mm. of Hg are plotted as points against the voltage $V = E_0 - R$ as abscissae in Figs. 1, 2, and 3. Neglecting for the



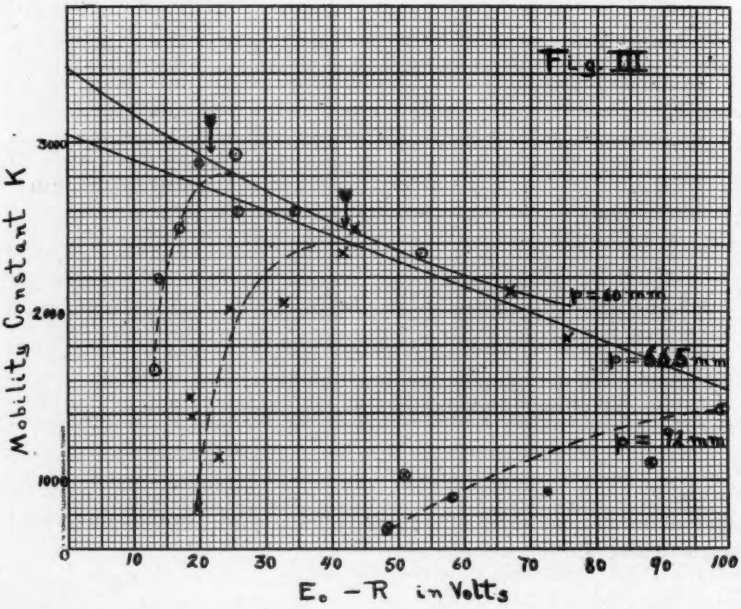
minute the sharp downward bends of the curves at 60 and 66.5 mm. the points as a whole lie closely on the smooth curves drawn in the figures which represent the following equations:

$$1.\ p = 41 \quad K = \frac{105{,}900}{8.8 + V}; \qquad 2.\ p = 51.5 \quad K = \frac{205{,}800}{39 + V};$$

$$3.\ p = 60.0 \quad K = \frac{244{,}000}{63.8 + V}; \qquad 4.\ p = 66.5 \quad K = 3074 - 15.4\,V,$$

In the last equation there is hardly enough of the curve obtainable before the break to warrant this extrapolation. In the case of the curve at 92 mm. the break occurs at values of V higher than it was experimentally possible to measure. As the highest point at 92 mm. lies pretty close to the continuation of the curve at 66.5 mm. one might assume that it would increase as does the latter mobility, were it not for the downward trend which is due to another cause.

The curves at once show that the values of K as a function of V at different pressures do not fall into a family of curves as is the case for the electrons in H_2 and N_2, and this irregular behaviour makes it also unwise to attempt to reduce the value of K obtained for $d = 1.955$ cm. to that for any other value. The cause for this peculiar behaviour of the curves is completely unaccounted for by the theory, and may be due to the effect of attachments. The attachments can interfere with the mobility in two ways. In the first case attachment may create an appreciable concentration of slow moving ions near the upper plate whose space charge must drive electrons and ions to that plate. This would result in giving ab-



normally high mobilities, as electrons would reach the upper plate at voltages below E_0. The occurrence of high mobilities ascribable to such fields was observed in the case of electron mobilities at 60 and 66.5 mm. pressure for values of V below the break points of the curves when the field R was absent. These fields it was assumed were reduced to negligible values, or were sufficiently neutralized by R to be ineffective in the measurements recorded. In the second case attachments might cause a sharp and rapid decrease in the electron mobilities. For if the average electron attached to a molecule within a distance less than d, the plate distance, the mobility would be markedly decreased as ions move but 2 cm./sec. per volt/cm.

That this is the cause of the sharp downward bends of the curves at 60 mm. and 66.5 mm. as V decreases, as well as the downward trend of the curve at 92 mm., will be shown by the evaluation of the constant n, the constant of attachment for air, from the data which the bends yield. This explanation is furthermore borne out by the results of some recent measurements which Dr. H. B. Wahlin has kindly communicated to the writer. In measuring the mobilities of electrons in N_2 at low fields (2 to 3 volts per cm.) Wahlin observed that the mobility curves showed a marked downward bend as the fields decreased, which were quite similar to those obtained by the writer at much higher fields in air. On more careful purification of his gas the decrease observed disappeared.

In order to evaluate n from the bends one must turn to the theory of attachment of J. J. Thomson.[4] The theory gives the number of electrons that attach between x and $x + dx$, as

$$dN = A\beta e^{-\beta x}\, dx, \quad \text{where } \beta = \frac{c}{\frac{E_0}{d} kn\lambda},$$

and A is the number of electrons which start at $x = 0$. Here λ is the electronic mean free path, c its velocity of thermal agitation, E_0/d the field strength, n the average number of impacts required of an electron before it is able to attach, and k is the electron mobility. The average distance which an electron goes without attaching is therefore given by

$$\Delta = \frac{\int_0^N x dN}{A} = \frac{1}{A}\int_0^\infty x\beta e^{-\beta x} dx = \frac{1}{\beta} = \frac{k\frac{E_0}{d} n\lambda}{c}$$

Now according to Townsend $k = 0.815e\lambda/mc$ and hence the equation for Δ becomes

$$\Delta = 6.8 \times 10^{-16}(k^2)\frac{E_0}{d} n = 6.8 \times 10^{-16}\left(\frac{760}{p}\right)^2 K^2 \frac{E_0}{d} n$$

Now at the point where K begins to decrease for a given pressure we have $\Delta = d$ and hence $\Delta d = d^2 = 6.8 \times 10^{-16} K^2 E_0 n(760/p)^2$. Furthermore at a pressure p,

$$k = \frac{\pi}{\sqrt{2}}\frac{d^2}{E_0 T}, \qquad \text{whence we get } d = \sqrt{\frac{\sqrt{2}}{\pi} E_0 TK \frac{760}{p}}.$$

Thus we can set

$$6.8 \times 10^{-16}\left(\frac{760}{p}\right)^2 K^2 E_0 n = \frac{\sqrt{2}}{\pi} E_0 TK \frac{760}{p}$$

whence $$n = \frac{\sqrt{2}}{\pi 6.8 \times 10^{-16}}\left(\frac{p}{760}\right)\frac{T}{K} = 6.61 \times 10^{-14}\,(p/760)\,T/K.$$

For the two well-defined breaks at 60 mm. and 66.5 mm. we have the following data: at $p = 60$ mm., $V_0 = 19.9$, $K = 2890$, $1/T = 81600$, whence $n = 2.2 \times 10^5$, and at $p = 66.5$ mm., $V = 43.3$, $K = 2440$, $1/T = 141{,}000$, whence $n = 1.69 \times 10^5$. For the break at 92 mm. one cannot evaluate n, as the location of the break is impossible. The best value obtained for n experimentally from the ionic mobility curves is about 2.0×10^5 for air.[5]

One is thus free to assert that the bends in the curves mark the beginning of the average attachment. In how much the attachments which occur within distances less than d when the average electron is still crossing without attachment affect these results cannot be estimated. One is then forced to accept the values of the electron mobilities in air given above as the only data which we have on these quantities above 20 mm. pressure. Before utilizing them in any computations it is however necessary to add one more correction. The mobilities of the electron were measured with an alternating current of sine wave form. As will be seen in a forthcoming paper on electron mobilities in He, the mobility K obtained from the experiments and expressed in an equation of the form $K = a/(b + V)$ must have its constant "a" divided by 1.57, and its constant "b" multiplied by 0.568 in order to give the true value of K. For an equation of the form $K = a - bV$, the "b" must be multiplied by $4/\pi$. Putting in these corrections, the true electron mobility constant (reduced to 760 mm., in air), in as far as it can be given by this method, is represented by the following equations.

$$p = 41 \text{ mm.} \qquad K = \frac{67{,}500}{5.0 + V}$$

$$p = 51.5 \text{ mm.} \qquad K = \frac{130{,}400}{221 + V}$$

$$p = 60.0 \text{ mm.} \qquad K = \frac{155{,}000}{36.3 + V}$$

$$p = 66.0 \text{ mm.} \qquad K = 3074 - 19.55V$$

$$p = 92.0 \text{ mm.} \qquad K = 3074 - 19.55V$$

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[1] Loeb, L. B., *Physic Rev.*, N. S., **19**, 1922 (24).

[2] Loeb, L. B., *Ibid.*, **20**, 1922 (398).

[3] Loeb, L. B., *Ibid.*, **17**, 1921 (89).

[4] Thomson, J. J., *London, Phil. Mag.*, 1915 (318).

[5] Loeb, L. B., *Physic Rev.*, N. S., **21**, 1923 (384).

THE THERMAL CONDUCTIVITY OF LIQUIDS

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This note briefly summarizes new results on the thermal conductivity of liquids which will be described in greater detail in a forthcoming number of the *Proceedings of the American Academy of Arts and Sciences*. The results are of two kinds: in the first place measurements of the effect of pressure on thermal conductivity, and in the second place a theoretical expression for thermal conductivity.

The apparatus used in the measurements was a radial flow apparatus, in which the liquid was contained in the space between two concentric metal cylinders; the axis of the inner cylinder was a source of heat which flowed out radially across the layer of liquid. The temperature difference between the outer and inner surfaces of the layer of liquid was measured, and the thermal conductivity calculated in terms of the dimensions and the temperature difference. The method is particularly adapted to the measurement of substances of low conductivity like liquids, and gave results of much regularity and consistency. It is adapted to give not only the changes of conductivity under pressure, but also the absolute values of conductivity. The accuracy of the results is probably a few tenths of one per cent.

Fifteen liquids were measured in all. Many of these were the same as those for which I have previously determined the various thermodynamic properties under pressure.[1] The pressure range of this work was that usual to all my high pressure measurements, namely 12000 kg./cm.^{2}; the measurements were also made at two temperatures, 30° and 75°. Not only do the measurements give the pressure coefficient of conductivity, which has never been measured before for liquids, but also the temperature coefficient at atmospheric pressure, for which there are only a few discordant previous measurements, and the temperature coefficient of the pressure coefficient.

The new experimental results are contained in table I. The results for each liquid are given in two lines, in the first line are the results for 30° and in the second those for 75°. In the first column of figures are the values at atmospheric pressure of the absolute conductivity (in ordinary units, gm. cal. per cm.^{2} per sec. per degree C. per cm.), in the second column the ratio of the conductivity at 6000 kg. to that at atmospheric pressure, and in the third column the ratio of the conductivity at 12000 kg. to that at atmospheric pressure. The general results may be summarized as follows. At atmospheric pressure the conductivity decreases with rising

TABLE I

SUMMARY OF RESULTS ON THERMAL CONDUCTIVITY OF LIQUIDS

LIQUID	K, ABSOLUTE CONDUCTIVITY	$\frac{K_{6000}}{K_0}$	$\frac{K_{12000}}{K_0}$
Methyl Alcohol	0.000505	1.724	2.097
	493	1.785	2.191
Ethyl Alcohol	430	1.744	2.122
	416	1.845	2.278
Iso-propyl Alcohol	367	1.743	2.150
	363	1.890	2.211
N Butyl Alcohol	400	1.648	2.008
	391	1.720	2.099
Iso-amyl Alcohol	354	1.686	2.069
	348	1.724	2.126
Ether	329	2.009	2.451
	322	2.043	2.537
Acetone	429	1.659	Freezes
	403	1.738	2.137
Carbon Bisulfide	382	1.663	1.962
	362	1.789	2.154
Ethyl Bromide	286	1.657	1.928
	273	1.772	2.121
Ethyl Iodide	265	1.509	1.724
	261	1.570	1.837
Water	144	1.293	Freezes
	154	1.308	1.506
Toluol	364	1.604	Freezes
	339	1.738	2.089
Normal Pentane	322	1.987	2.481
	307	2.112	2.740
Petroleum Ether	312	1.970	2.379
	302	2.026	2.561
Kerosene	357	...	...
	333	1.654	2.054

temperature, except for water. At constant temperature the conductivity increases with rising pressure, the increase not being linear with the pressure, but the increase in the first 6000 kg. is relatively greater than in the second. The total increase under 12000 kg. varies from 1.5 to 2.7 fold, the increase being greater for the more compressible liquids. The proportional increase is greater at the higher temperature.

The second part of the paper has to do with a very simple theoretical picture in terms of which the conductivity may be calculated. It is surprising that practically no theoretical work has been done on the thermal conductivity of liquids. I know of no theoretical expression for the conductivity, and I have been able to find only one empirical expression, that of Weber.[2] It is known in a general way that the mechanism of conduction in a liquid is different from that in a solid or a gas. That the mechanism is different from that of a solid is shown by the fact that the con-

ductivity is of a lower order of magnitude. That it is different from that of a gas is shown by the fact that the theoretical results for a gas do not hold. Thus the well known relation between thermal conductivity, viscosity, and specific heat of a gas breaks down entirely for a liquid. It has been shown, for example, that the viscosity of a dilute solution of glue in water may vary by a factor of many fold with very little change in the thermal conductivity.

In searching for a better understanding of the mechanism of thermal conduction in liquids, I have found the following expression for conductivity in terms of fundamental properties of the liquid:

$$K = 2\alpha v \delta^{-2}$$

Here k is the thermal conductivity measured in Abs. C.G.S. units, α is the gas constant 2.02×10^{-16}, v is the velocity of sound in the liquid, and δ is the mean distance of separation of the centers of the molecules, computed by assuming the molecules arranged in simple cubical array.

TABLE II

COMPARISON OF COMPUTED AND OBSERVED THERMAL CONDUCTIVITIES

LIQUID	VELOCITY OF SOUND	δ^{-2}	THERMAL CONDUCTIVITY COMPUTED	THERMAL CONDUCTIVITY OBSERVED
Methyl Alcohol	1.13×10^{5}	6.00×10^{14}	27.4×10^{3}	21.1×10^{3}
Ethyl Alcohol	1.14	4.74	21.8	18.0
Propyl Alcohol*	1.24	3.94	19.7	15.4
Butyl Alcohol*	1.05	3.49	14.9	16.7
Iso-amyl Alcohol	1.24	3.13	15.7	14.8
Ether	.92	3.19	11.9	13.7
Acetone	1.14	4.00	18.5	17.9
Carbon Bisulfide	1.18	4.61	21.9	15.9
Ethyl Bromide	.90	3.97	14.5	12.0
Ethyl Iodide	.78	3.81	12.1	11.1
Water	1.50	10.4	63.0	60.1

* Calculations made for normal propyl and isobutyl alcohols, observed conductivities given for iso-propyl and normal butyl.

In table II are shown the values at 30° computed by the formula and determined by experiment for those liquids for which the necessary data are known. In making the computation, I have had to calculate the velocity of sound from my values of isothermal compressibility, making correction for the difference between isothermal and adiabatic compressibility, a difference which may rise to as much as 30 or 40%. There is probably some error in these values (my values for the thermodynamic constants at atmospheric pressure had to be determined by extrapolation), but in the absence of systematic measurements of the velocity of sound in liquids, it seems the best that we can do.

The formula may be deduced in terms of the following very simple physical picture. Let there be in the liquid a temperature gradient $d\theta/dx$. The energy of the molecule is $2\alpha\theta$ (half potential and half kinetic), where

θ is the absolute temperature. The difference of energy between neighboring molecules in the direction of the temperature gradient is $2\alpha\delta d\theta/dx$. This energy difference is to be conceived as handed down a row of molecules with the velocity of sound. The total energy transferred across a fixed point of any row of molecules per unit time is the product of the energy difference and the number of such energy steps contained in a row of molecules v cm. long, or $2\alpha\delta(d\theta/dx)(v/\delta)$. The total transfer across unit cross-section is the product of the transfer across a single row and the number of rows in unit cross-section, or $2\alpha v\delta^{-2}d\theta/dx$. But by the definition of thermal conductivity the transfer is also $kd\theta/dx$. Whence, identifying constants, we have for the thermal conductivity $2\alpha v\delta^{-2}$.

We see from table II that the agreement between theoretical and experimental values is much more than an agreement of order of magnitude, and is striking in view of the simplicity of the formula and the fact that this appears to be the first theoretical formula ever applied to the subject. It is interesting that the conductivity of water, which is three or four times as high as that of the ordinary organic liquid, is reproduced by the formula. The high conductivity of water appears, therefore, not to be due in any special way to the molecular peculiarities of water (two or more molecular species), but is directly referable to its low compressibility and the fact that the centers of the molecules in water are closer together than in the ordinary liquid.

The formula gives the right sign for the temperature coefficient of conductivity at atmospheric pressure, both for ordinary liquids and water. For the ordinary liquid, both v and δ^{-2} decrease with rising temperature, so that the conductivity would be expected to decrease with rising temperature, as it actually does, whereas for water v increases with rising temperature (both the isothermal compressibility and the thermal expansion of water vary abnormally with temperature) at a rate more than sufficient to compensate for the decrease of δ^{-2}, so that on the whole the conductivity should increase, as it does in fact.

The formula does not account well numerically for the effect of pressure. It of course gives the right sign for the pressure effect, for both v and δ^{-2} increase under pressure. But the increase of these two factors under pressure is such that an increase of conductivity under 12000 kg. of between three and four fold would be expected, whereas we have found that the actual increase is only 2.7 fold at the maximum. The reason for this failure is not far to seek. Our theoretical picture has represented the molecules of the liquid as standing in more or less coherent rows, so that the temperature energy may be handed from molecule to molecule down a row with little interference. It is not unnatural to suppose that the molecules tend to align themselves in this way when the intermolecular forces are allowed free play, for it is known that in the surface films of many

liquids the molecules do form into rows. But when external compulsion is applied, as by the application of external pressure, the molecules are no longer able to take their natural arrangement, but are forced into less natural positions, the coherent rows are broken up, the transfer of energy is interfered with, and the thermal conductivity is therefore less than to be expected according to the simple picture. This agrees with the observed fact that the increase of conductivity under pressure is less than that given by the simple formula.

Further refinement of the picture would obviously demand a detailed knowledge of molecular structure and of the fields of force surrounding the molecules.

[1] Bridgman, P. W., *Proc. Amer. Acad., Boston*, **49**, 1913 (1–114).

[2] Weber, H. F., *Sitzber. Ber. Akad., Berlin*, 1885[2] (809–815).

PHOTOVISUAL MAGNITUDES OF ONE HUNDRED BRIGHT STARS

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The combined results of my extra-focal observations for the photographic magnitudes of the brighter stars are given in *Harvard Annals*, **76**, No. 6. According to this system, the photographic magnitudes were made to correspond to Professor Pickering's photometric magnitudes for stars of Class A0.

The color indices, derived by subtracting the photometric from the photographic magnitudes, are in good agreement with the results of Parkhurst[1] and Schwarzschild.[2] Parkhurst used a Zeiss doublet lens of ultraviolet glass; Schwarzschild used a Zeiss Tessar lens; while I employed three different instruments, namely, the 11-inch Draper, the 13-inch Boyden, and the 8-inch Draper telescopes. Parkhurst placed his plates 0.6 cm. inside of focus; Schwarzschild placed his plates from 0.05 to 0.10 cm. inside of focus. All of my plates have been 1.25 cm. or more outside of focus. The accordance of the several results is probably due to the fact that extra-focal images partake but little of the idiosyncrasies of the individual telescope. Herein they have great advantage over focal images. Moreover, all the instruments were refractors. I have shown elsewhere that color indices found with the 24-inch Harvard Reflector[3] are less than those obtained with the 8-inch Draper telescope. This is due to the selective reflectivity of a silvered surface.

The present investigation concerns the *photovisual* magnitudes of bright

stars and is being published in detail in *Harvard Annals*, **85**, No. 3. A preliminary discussion of the results for 24 stars has been given in *Harvard Annals*, **81**, No. 4. One hundred stars have now been observed with the 8-inch Draper telescope. Cramer Isochromatic Instantaneous plates have been used with a yellow filter, which cuts off all of the blue light. New apparatus was provided for setting the plate at different distances from the focal plane; but it was found that for a lens of large angular aperture that various inaccuracies, particularly for positions nearest to focus, may introduce slight errors, unless special precautions are taken. Even the curvature of the photographic plates may affect the results. A study was made by placing over the objective a grid of parallel wires arranged at intervals of 2.0 cm., and photographing out-of-focus images of a star at *different distances from the focus*. From measures of such images it is possible to determine the focal plane and the focal length of the objective.

To provide against errors due to the inaccuracies of the settings, I have, in observing, used a special cap over the objective, which has two wires parallel to each other and 16.0 cm. apart. The distance between the images of the wires, as shown on the out-of-focus disk, has been measured for nearly all the observations of this investigation. The other dimensions of each disk also have been measured and correction made for diffraction. From all these measures the light-value for each image is found. In other respects, the methods of observation, measurement, and reduction are the same as I have used heretofore.

Photovisual magnitudes of one hundred of the brightest stars, observable at Cambridge, have been determined from observations made on not fewer than five nights for each star. These magnitudes agree with the photometric magnitudes for stars of Class A0. The average deviation of the results for each star from the mean of the five or more observations is about ± 0.07 magnitudes. The photovisual magnitude of Polaris is found to be 1.99, corresponding to the photographic magnitude of 2.69, and the photometric magnitude of 2.12.

The stars are distributed, according to class of spectrum, as follows: B, 23; A, 32; F, 8; G, 9; K, 21; and M, 7. The following table gives the differences, photovisual magnitude minus photometric magnitude.

CLASS	NUMBER	DIFFERENCE	CLASS	NUMBER	DIFFERENCE
B0	6	−0.05	F0	2	+0.03
B1	3	+0.06	F5	4	−0.09
B2	3	−0.08	F8	2	−0.14
B3	2	+0.02	G0	8	−0.15
B5	2	−0.01	G5	1	−0.21
B8	7	−0.07	K0	14	−0.16
A0	16	0.00	K2	4	−0.18
A2	5	−0.06	K5	3	−0.16
A3	3	−0.05	M	7	−0.19
A5	8	−0.04			

The above values may be used to transform photometric to photovisual magnitudes. Perhaps, a better plan is to use general corrections according to spectral class, as follows: B, −0.02; A0, 0.00; A2, A3, and A5, −0.05; F, −0.10; G, −0.15; K, −0.16; and M, −0.20. Comparing photovisual magnitudes, computed thus, with the observed photovisual magnitudes of our list of stars, we find an average deviation of ±0.07 magnitudes.

The coefficient of photovisual atmospheric absorption is determined at 0.25, in contrast with the value of 0.47 for the absorption coefficient of the photographic rays.

The color indices for the several classes of spectrum are as follows: B0, −0.23; B1, −0.17; B2, −0.26; B3, −0.20; B5, −0.10; B8, −0.08; A0, −0.02; A2, +0.13; A3, +0.18; A5, +0.21; F0, +0.25; F5, +0.59; F8, +0.80; G0, +0.88; G5, +1.16; K0, +1.28; K2, +1.61; K5, +1.73; and M, +1.87. These values are independent of any visual observations. In general, the photovisual color index is greater than the visual or the photometric color index.

Observations have been begun also at Arequipa for the photovisual magnitudes of a selected list of southern stars. This work, when complete, may introduce some slight changes.

Correction has been made for the variation of Polaris, which was used almost exclusively as the comparison star in the present investigation. The residual curves exhibit errors for the plate or the night, which seem to indicate the presence of some recurring factor. Whatever may be the cause, the corrections actually used for the plate or night are the best to be derived from the data, since they include not only the peculiarities of Polaris, but any change in the conditions occurring between the times of photographing Polaris and the other stars on the same plate.

The color index of Nova Aquilae No. 3 was found in 1918 from photovisual and photographic magnitudes determined by the out-of-focus method.[4] The photovisual magnitude of Altair, the comparison star, was not then well determined. According to the latest value, which is 0.72, the photovisual magnitudes found for the nova should be corrected by 0.16 magnitudes, and the color index is changed from −0.35 to −0.19. This change brings it into accord with the color index found by comparing the photographic with the visual observations made at that time.

[1] *Astrophys. J., Chicago, Ill.*, **36**, 1912 (169–227), p. 218.
[2] *Göttingen Aktinometrie, Teil B*, 1912 (1–81), p. 27.
[3] *Harvard Annals, Cambridge, Mass.*, **76**, No. 10, 1915 (175–189).
[4] *Ibid.*, **81**, 1919, p. 213.

REVISED MAGNITUDES AND COLOR INDICES OF THE PLANETS.

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In connection with the preceding investigation relating to the photovisual magnitudes of the stars, photovisual and photographic magnitudes of the brighter planets have been determined by out-of-focus methods. These observations were made with the view of obtaining better values of color index. The detailed discussion will appear soon in *Harvard Annals*, **85**, No. 4.

The number of plates taken for this purpose, including those reported in *Harvard Annals*, **84**, No. 4, is 130. A summary of the results from all these observations is contained in the following table, which gives magnitudes for Venus at superior conjunction, and of the other planets at mean opposition, together with the color index of each.

PLANET	PHOTOGRAPHIC MAGNITUDE	PHOTOVISUAL MAGNITUDE	COLOR INDEX
Venus	−3.11	−4.02	+0.91
Mars	−0.55	−2.00	+1.45
Jupiter	−1.63	−2.59	+0.96
Saturn (without the rings)	1.87	0.65	+1.22
Uranus	6.25	...	+0.74

The relation between magnitude and phase has been derived as far as possible. For Venus, the following formula has been found, in which α is the phase angle, and -4.291 is the photographic magnitude for Venus referred to its mean distance from the Sun and unit distance from the Earth:

$$\text{Photographic Magnitude} = -4.291 + 0.01445\alpha + 0.0000002251\alpha^3.$$

Computing, by means of the above formula, the photographic magnitudes for the phase angles of 15 photovisual observations of Venus, and comparing them with the corresponding observed magnitudes, we find a mean value of $+0.91$ for the color index. Care was taken that on some nights both photographic and photovisual observations should be made consecutively. These give color index directly, freed from error in phase angle and distance, and also to a large extent from error due to atmospheric absorption. The color index of Venus found from 7 pairs of such observations is $+0.91$, the same as derived by use of the formula.

For Mars the following linear formulae are found:

$$\text{Photographic Magnitude} = -0.55 + 0.0202\alpha$$
$$\text{Photovisual Magnitude} = -2.00 + 0.0152\alpha.$$

Values of the color index of Mars, derived from pairs of plates made on 15 nights, give a mean of +1.40. The value of +1.45 given by the above formulae has been accepted.

For Jupiter the effect of phase, of which there were but slight indications, was disregarded and the means of all the observations have been taken. These give a color index of +0.96. Comparison of pairs of plates for 12 nights shows a mean of +0.97.

The reduction for Saturn is involved with the inclination of the rings as well as the phase. The phase constant was found from the observations obtained in 1921 and 1922, when the inclination was small. The effect of the inclination of the rings on the observed brightness of the planet is well represented as proportional to the sine of the inclination. On this hypothesis, the average deviation of the values of eight years is ±0.03 magnitudes. The complete formula for Saturn follows, in which the inclination B, is considered always positive:

$$\text{Photovisual Magnitude} = 0.653 - 1.626 \sin B + 0.0419\alpha.$$

Comparing 14 computed photovisual magnitudes of Saturn with the photographic magnitudes actually observed gives a color index of +1.24. Comparison of pairs of plates for 13 nights gives +1.21. The value accepted for the color index is +1.22.

For Uranus the photographic magnitude is the mean of all the observed values. The color index is found by comparison with the photometric magnitude given in *Harvard Annals*, **46.**

From the preceding, it will be seen that the color indices found from the formulae involving phase are confirmed by comparison of photovisual and photographic observations obtained consecutively on the same nights.

THE ORIGIN OF SECRETORY GRANULES

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Secretions commonly make their first appearance in the cytoplasm of variously specialized gland cells in the form of granules which undergo a transformation into the active components of the secretion (enzyme, etc.), at or shortly before their expulsion from the cell. The origin of these secretory granules has long been recognized as a matter of great interest. They were at first thought to arise from the nucleus, but little evidence in support of this view has been found. Various formed elements of the cytoplasm have also been regarded as the immediate fore-runners of the secretory

granules, and in particular the possibility of a transformation of mitochondria into secretory granules has recently gained serious consideration. As yet, however, none of these explanations has gained general acceptance, and the problem is still far from a final solution.

The recent revival of interest in the Golgi apparatus has directed attention to this component of the cell as possibly involved in the synthesis of secretory products. In a recent paper by Nassonov,[1] a convincing demonstration has been given of a close association between the strands of the Golgi apparatus and the early secretory granules, an association which, however, is soon lost, the growth of the granules being completed independently. Such granules are accompanied by a girdle or cap of material which behaves micro-chemically like the Golgi apparatus and is interpreted by Nassonov as a piece of Golgi material which each granule carries away upon the severance of its original connection with the centralized apparatus.

These observations were made on several glands of the salamander. I have since carefully studied the question in similar material, extending Nassonov's observations with results that are in remarkable accord with his. The Golgi apparatus in these gland cells occurs in the form of a delicate, anastomosing network, along the strands of which the smaller secretory granules are ranged in what appears to be a very intimate relationship. There is abundant evidence from the results of both Nassonov and myself, that the granules on leaving the Golgi apparatus actually are provided with a miniature replica of the Golgi material in the form of a small cap, which is closely applied to the periphery of the granule—a point of great importance. These facts taken together clearly indicate a connection of some kind between the Golgi apparatus and secretory phenomena. Furthermore, my observations, like those of earlier workers on the behavior of the Golgi apparatus in cells where secretion is more or less intermittent or cyclical in character, demonstrate that at the time of secretory activity the Golgi material undergoes decided morphological changes. I am inclined to conclude from these observations that the changes in the Golgi apparatus are connected with secretory activity as a cause rather than a result. In what way may this relation between the Golgi apparatus and secretory granules be interpreted?

Past failures to solve the origin of the secretory granules have certainly been due in part to the large numbers of such granules which occur in a single cell, and to the fact that they very early leave their place of origin and move into distant parts of the cell, where their growth is completed. This suggests that the mechanism through which a secretory granule is developed must be capable of operating in any part of the cell where such granules are visible. If we could find a secretory product consisting of a single granule, the study of its origin and growth would be greatly facilitated, and the results thus obtained might reasonably be extended to gland-cells of the usual

complicated type. Such a secretory product has, as a matter of fact, been found in the acrosome of the animal sperm, the supposed function of which as a mechanical boring device is becoming increasingly improbable. If we can divest our minds of the old ideas which have long been associated with the acrosome, we may be in position to develop a new interpretation which seems to me more promising.

The acrosome, as shown by a number of recent investigations of the sperm cells, is always developed in association with the Golgi apparatus. In a typical case like the mammal or salamander, the acrosome arises as a vesicle which remains in close connection with the Golgi apparatus throughout its period of formation, and from it the Golgi apparatus itself is finally separated, to be cast out of the sperm apparently as a useless remnant. The differentiation of the acrosome is thus in some way related to the Golgi apparatus, though not in the sense of a direct transformation of the one into the other. I venture to suggest an analogy between the acrosome and a secretory granule applied to the head of the mature sperm, whence its substance may be released at the time of fertilization and thus parallel the fate of many other secretory granules. It even seems possible that the acrosome may be thought of as an enzyme-bearer comparable to other secretory products, whose function it is to set off the fertilization reaction. The relation of such an interpretation to the fertilizin theory of F. R. Lillie will be obvious. With such a possibility in mind it becomes of interest to examine more closely the mode of formation of the acrosome, as offering a possible clue to the origin of secretory granules in general. Since the acrosomal vesicle may make its first appearance within the Golgi complex of the spermatid, it seems probable that the acrosomal material is not merely condensed on the surface of the Golgi apparatus, but is in some way a redifferentiation product of the Golgi materials themselves. The Golgi apparatus would thus be a synthetic center in which or from which the acrosomal materials are "secreted."

If we may extend these considerations to the obscure conditions seen in the gland-cell, we arrive at a conception somewhat as follows. In gland cells the secretory granules arise within or on the Golgi apparatus, the exact method of origin being in doubt, but presumably comparable to that of the acrosome. These secretory granules are soon detached from the Golgi reticulum and migrate into other parts of the cell where their growth is completed. Each granule, however, carries with it a fragment of the Golgi material, and its growth is thus provided for quite independently of the main mass of the Golgi apparatus, which remains as a source of primary origin. The independent granules with their attached bits of osmiophilic substance have a striking counterpart in the lepidopteran spermatid, where the acrosome arises from multiple vesicles each of which develops in connection with a separate portion of the Golgi apparatus. It appears, fur-

thermore, from the old observations of Heidenhain,[2] that there may be something in the secretory process comparable to the casting off of the Golgi apparatus subsequent to acrosome formation, the secretory granule then transforming into the definitive secretion product.

If these comparisons have any weight they may serve to call attention to the possibility of a direct relation of the Golgi apparatus to processes of chemical synthesis, and in particular those of enzyme formation.

[1] Nassonov, D. N., *Arch. mik. Anat.*, **97**, 1923 (136–186).

[2] Heidenhain, M., *Jena*, **1907**, 506 pp.

THE HEREDITY OF MICROSCOPIC HAIR CHARACTERS IN PEROMYSCUS

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An investigation which consisted of (1) working out a method of quantitatively measuring certain hair characters producing coat color in the deer-mouse, and (2) crossing two geographic races in a species (*Peromyscus eremicus*) not hitherto bred in captivity, is now in a position warranting a first report. Preliminary work was done on two racial crosses of *P. maniculatus: P. m. rubidus* from Eureka and Carlotta, California, with *P. m. sonoriensis* from Victorville, California. Previous results of the crossing of these so-called subspecies have been published by Dr. F. B. Sumner,[1] under whose direction the present work was prosecuted. Using another species a rather more complete study was made of a larger number of hybrids resulting from a cross between *P. e. eremicus* from Death Valley, California, and *P. e. fraterculus* from La Jolla, California. Altogether 647 racial hybrids in the two species were sampled.

Four classes of hairs which varied in proportion and intergraded, could be identified in a sample from any individual; such classes differing in both the pigmentation and size of the contained hairs. The differences in the proportions of the various classes served both to differentiate the races crossed, and to give a quantitative measure of color difference, individual or racial. These classes were called A, B, C, and D. The number of hairs in each class was expressed as a percentage of the total number in the sample, this number being obtained by counts under the microscope.

Besides the characters obtained by counts 3 others were obtained by measurement, viz: the total length of the hair at the point sampled, the

length of the subterminal agouti band, and the length of the dark terminal tip; such measurements being confined to one class of hairs, and applied fully to one racial cross.

Finally there were characters which if present were restricted to either one race or the other.

Different races differed, as a rule, in the mean value of all the characters used (there was one exception to this) and there was a certain parallelism in the difference between coast and desert races in both species. That is, a coast race differed from a desert race in the species *maniculatus* in most of the ways in which a coast race differed from a desert race in the species *eremicus;* and this was the case in characters strongly, as well as ones not strongly correlated with color differences. The assumptions (1) that all the differences thus observed between the contrasted races have been evolved in the wild state, and (2) that some at least represent an actual effect of the environment, direct or indirect, are therefore warranted. This latter supposition is strengthened by the fact that like measurements of the subspecies *P. m. gambeli* give values intermediate between those obtained in the subspecies *rubidus* and *sonoriensis*.[2]

TABLE I

A COMPARISON OF THE COEFFICIENTS OF VARIABILITY OF THE CHARACTERS MENTIONED

RACIAL CROSS	GENERATION	NO.	A	B	C	D
Carlotta-Victorville	F_1	40	5.94±.45	28.29±2.30	53.64±5.08	36.08±3.05
Carlotta-Victorville	F_2	118	7.70±.34	38.07±1.90	62.36±3.65	46.84±2.47
Difference			1.76±.56	9.78±2.98	8.72±6.26	10.76±3.92
Eureka-Victorville	F_1	53	4.45±.29	28.80±2.04	43.63±3.35	38.80±2.90
Eureka-Victorville	F_2	90	6.43±.32	39.66±2.28	51.53±3.20	41.30±2.39
Difference			1.98±.43	10.86±3.06	7.90±4.63	2.50±3.76
P. eremicus-fraterculus	F_1	199	8.39±.28	15.87±.55	29.95±1.10	32.00±1.19
P. eremicus-fraterculus	F_2	147	9.74±.38	16.41±.66	33.91±1.48	33.01±1.43
Difference			1.35±.47	.54±.86	3.96±1.84	1.01±1.86

RACIAL CROSS	GENERATION	NO.	TOTAL LENGTH OF HAIR	LENGTH OF AGOUTI BAND	LENGTH OF DISTAL TIP
P. eremicus-fraterculus	F_1	199	6.00±.20	10.68±.36	9.96±.34
P. eremicus-fraterculus	F_2	147	6.90±.27	16.48±.67	12.85±.51
Difference			.90±.34	5.80±.76	2.89±.61

NOTE: A, B, C, and D refer to classes of hairs obtained by counts, the number in each class being expressed as a percentage of the total number in a sample. Total length of hair, length of agouti band, and length of tip were measured. Carlotta-Victorville and Eureka-Victorville are each a *rubidus-sonoriensis* cross.

The results of all three racial crosses mentioned are quite consistent in trend and may be briefly summarized as follows:

(1) The characters in question were shown to be hereditary by the effect of selection,[3] and by parent-offspring correlations.

(2) Both the first and second hybrid generations exhibited mean values for a character, intermediate as a rule between the parental means, though the mean values for the two hybrid generations differed at times.

(3) The F_2 generation was consistently more variable than the F_1 generation, as may be seen in table I. The differences are more significant than they appear, for they were evident in spite of there being no inbreeding of the parental races, and in spite of a rather small sample of hairs in the counted classes. It could be shown that this increase in variability was probably not due to any non-genetic circumstance affecting particularly the F_2 generation.

(4) Of three pigmental characters (not included in the table) which were introduced into the *eremicus-fraterculus* cross by certain individuals of either one parent race or the other, one character reappeared only in the F_2 generation, and the other two in a greater proportion of the F_2 generation.

(5) F_2 sibs were shown by both parent-offspring and fraternal correlations to be rather more unlike each other than F_1 sibs. This in spite of some mating of F_1 sibs (assortative mating).

(6) The plotted curves of the two hybrid generations were rather characteristically different, those of the F_2 being as a rule shorter and flatter topped, or in some cases slightly indented.

These results, a full report of which is to be presented later, are in some ways very similar to those already published on sub-specific hybrids in *Peromyscus*, but the ones here given so consistently meet the expectations of the multiple factor theory that the writer accepts it as the explanation of them.

It is suggested, in view of these later results, together with the tendency observable throughout previous work for the F_2 generation to be more variable, that a multiple factor explanation can reasonably be applied to the results of subspecific crosses in *Peromyscus* as a whole with perhaps the exception of differences in asymmetry. Ostensibly unconformable cases might be briefly summarized as:

(1) Cases in which the F_1 generation was equally variable, or even more variable than the F_2.

(2) Cases in which characters were seemingly the same in the two parent races, but in which an increase in variability in the F_2 generation was manifest.

(3) The cases of the increased variability in the F_2 generation with regard to the sinistro-dextral ratio of the paired bones, although asymmetry as such appears not to be inherited.

Together with the acknowledgment that at present the case cannot be proved for or against the presence of multiple factors, it is submitted that,

(1) Considering the fact that these races cannot be inbred prior to crossing, and that they cannot be, with present facilities reared in a perfectly uniform environment, some cases of decreased variability in the

F_2 would be quite expected. This would be especially likely with regard to characters having a presumably complex factorial representation, particularly when a small number of mice is used.

(2) Similarity of measurement in somatic characters in two different races is not a good enough criterion of a similar genetic composition to make the apparent segregation with regard to such characters an obstacle to a multiple factor explanation.

(3) The relevance of increased asymmetry to the question of multiple factors can only be determined by a more critical knowledge of the underlying cause of asymmetry. Crossing of races appears to have the effect of increasing asymmetry, especially in the F_2, but a cross of more distantly related races had less effect than the crossing of a more nearly related one; while statistical differences in asymmetry could be observed between generations of "pure" races.

It is realized that our knowledge of *Peromyscus* is in a condition that permits of only a tentative application of a particular theory. It is maintained, however, that many of the results which follow the crossing of wild races in this genus so well simulate the results expected, were multiple factors involved, that a Mendelian form of inheritance seems very probable.

[1] These PROCEEDINGS, **9**, No. 2, pp. 47–52, Feb., 1923.
[2] See also Sumner, F. B., *Amer. Naturalist*, **57**, May-June, 1923.
[3] Using material prepared by Dr. H. H. Collins.

PRELIMINARY NOTE. ON THE VARIATIONS OF THE SUN'S VISIBLE FEATURES ASSOCIATED WITH VARIATIONS OF SOLAR RADIATION

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For many years many observatories have regularly photographed the sun's direct image, and also the hydrogen and calcium clouds or flocculi revealed by the spectroheliograph. Comparatively little use has been made of these data for any quantitative purpose other than recording the march of the sun-spot curve. Within the past five years fairly accurate determinations of the intensity of solar radiation outside our atmosphere, usually called the solar constant, have been secured on a majority of days by the Smithsonian Institution at one or both of its solar radiation observatories.

Being courteously offered facilities for the examination of the collection

of prints of direct solar photographs and Hα spectroheliograms at the Mount Wilson Observatory, I have sought to find some connection between the variations of the sun's radiation and the sun's appearance. Nothing quantitative has yet been attempted, but I believe the four following general principles will be found to hold for almost all cases:

1. When sun-spots form or grow, or are brought by rotation into view on the visible disk, higher radiation values occur on the same day.

2. When sun-spots transit across the central diameter of the visible disk, lower radiation values occur, and usually reach a minimum on the day following the transit.

3. When the direct photographs and Hα spectroheliograms show a much disturbed solar surface, the radiation values run high.

4. When a period of quiescence in solar activity occurs, the radiation values tend to fall continually lower and lower until some new outbreak of activity is observed.

So well-supported are these four principles that I believe it will be possible to give them quantitative application, and possibly even to go so far in that direction as to establish a formula of several numerical terms by means of which the solar constant values may be computed approximately from photographic solar observations. The purpose of this note will be served if it induces directors of observatories where collections of such photographic solar observations are lying unused to cause them to be studied for such an object.

As for the reasonableness of the four principles, I doubt if it is necessary to argue in respect to numbers 1, 3 and 4. For just as an open fire in the grate is caused to give out a warmer glow if one stirs it and brings fresh coals to the front, so the sun should present a more intensely radiating surface whenever an increase of its convecting currents takes place. Such increases of convective activity are of course indicated to us by sunspots, faculae and flocculi. On the other hand when there is no such stirring of the sun's outer layers, and convection slackens, the enormous output of radiation must tend to cool the radiating surface, and the solar constant falls.

The principle number 2 deserves a fuller explanation. Readers of Volume IV of the *Annals of the Astrophysical Observatory* may recall that the observations of Guthnick on the brightness of Saturn indicated certain small fluctuations. These quantitatively agreed in percentage with the variations of the solar constant provided a time allowance was made to take account of the rotation of the sun from a position facing the earth to one facing Saturn. It seemed, in short, as if rays of a certain intensity of radiation, going out from the sun, rotated along with it, and so affected the different planets in the order of their heliographic longitudes.

If this is the case it would naturally be supposed that such rays would

stand nearly radially to the sun. But they might very well be inclined a little backwards so as to reach the earth a day or two later than the times of central presentation of their bases on the sun's surface.

Now it is often remarked in photographs of total solar eclipses that coronal rays seem to arise from sun-spots as their bases. In view of these considerations, what is more natural than to assume rays of diminished transparency extending out nearly, but not quite, radially from a sun-spot group? Thus the deepest obscuration of the sun viewed through such a veiling ray would occur a day or so after a sun-spot passed the sun's central meridian. Not all such rays would be equally inclined, so that the minimum of radiation might fall on the day of transit or even on the day before it.

Readers of *Annals*, Vol. IV, may also recall that a very conspicuous case of such a close coincidence of a sun-spot transit and a low solar constant occurred in March, 1920. Another hardly less conspicuous case occurred in May, 1921. In each case the minimum of radiation fell a day after the central transit of the most strongly spotted area. I now find the phenomenon to be nearly unfailing whenever a spot group goes by the sun's central meridian.

So much for the reasonableness and verification of the four principles. The first two taken together explain an obscure feature of observation stated in *Annals*, Vol. IV. It was found very certainly that high solar constant values prevail in years of sun-spot maximum. In other words, with increased activity there goes increased radiation. But if daily sun-spot numbers were compared with corresponding radiation values within any one year no such simplicity appeared. Sometimes low radiation attended very high sun-spot numbers. I now think it probable that if, in preparing the statistics, all days had been excluded when sun-spot groups lay between the two situations, one day previous and three days following central transit, this contradiction would have disappeared, and it would have been found consistently that the higher the solar activity the higher the solar radiation.